PHY 540. Statistical Mechanics.
Spring 2005. Solution of Homework #6.1

At T=0:

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2}{gv}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{6\pi^2 n}{g}\right)^{2/3};\ \overline{E} \approx \frac{3}{5}N\varepsilon_F;\ P = \frac{2}{3}\frac{\overline{E}}{V} \approx \frac{2}{5}n\varepsilon_F = \frac{\hbar^2}{2m}\frac{2}{5}n\left(\frac{6\pi^2 n}{g}\right)^{2/3}$$

At equilibrium $P_{1/2} = P_{3/2}$ $\Rightarrow \frac{2}{5}n_{1/2}\frac{\hbar^2}{2m}\left(\frac{6\pi^2 n_{1/2}}{g_{1/2}}\right)^{2/3} = \frac{2}{5}n_{3/2}\frac{\hbar^2}{2m}\left(\frac{6\pi^2 n_{3/2}}{g_{3/2}}\right)^{2/3}$

$\Rightarrow \frac{n_{1/2}}{n_{3/2}} = \left(\frac{g_{1/2}}{g_{3/2}}\right)^{2/5}$ since g_s=2s+1 $\Rightarrow \frac{n_{1/2}}{n_{3/2}} = \left(\frac{1}{2}\right)^{2/5}$

At $T \to \infty$ the gases behave as classical gases so:

$$P \approx nkT \qquad \Rightarrow n_{1/2}kT = n_{3/2}kT \Rightarrow \frac{n_{1/2}}{n_{3/2}} = 1$$